

Approximate Mean Time to Loss of Lock for the Symbol Synchronizer Assembly

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An approximate analysis of the mean time to loss of lock for the Symbol Synchronizer Assembly (SSA) is given. The results are used to determine the minimum required SSA input signal-to-noise ratio when the SSA is constrained to operate in the wide-wide or wide-medium bandwidth because of instabilities in the symbol stream time reference.

I. Introduction

The Symbol Synchronizer Assembly (SSA) is capable of operating at symbol signal-to-noise ratios as low as -5 dB (Ref. 1). This requires that the input symbol stream have a highly stable time reference so that after acquisition the SSA loop bandwidth can be narrowed for tracking. If the symbol stream time reference is less stable (for example, during playback of recorded Subcarrier Demodulator Assembly output) a wider SSA loop bandwidth must be used. This results in performance degradation due to the lower loop signal-to-noise ratio.

To determine the effect of bandwidth on Symbol Synchronizer Assembly lock, performance tests were performed at Goldstone (DSS 12) by R. Bartolett and R. Caswell. An input symbol stream with stable time reference was applied from the Subcarrier Demodulator Assembly and the Symbol Synchronizer Assembly constrained to operate with a fixed bandwidth setting. The results indicate that for 2-kbps (32,6) block coded data a signal-to-noise ratio $ST_b/N_0 \geq 8$ dB is required to hold lock in the wide-wide bandwidth ($w_L T_s = 0.02$) and $ST_b/N_0 \geq 4$ dB is required in the wide-medium bandwidth ($w_L T_s = 0.005$). In this article an approximate

analysis of the mean time to loss of lock is developed. The results compare favorably with the experimental data.

II. Analysis

The rms phase error for an in-lock linear phase-lock loop is given by

$$\sigma = \frac{1}{\sqrt{\text{SNR}_L}} \quad (1)$$

where SNR_L is the loop signal-to-noise ratio. For the SSA this is related to the symbol signal-to-noise ration $R = ST_s/N_0$ by (Ref. 2)

$$\text{SNR}_L = \frac{R \left[\text{erf} \left(R \frac{1}{2} \right) \right]^2}{\pi^2 W w_L T_s} \quad (2)$$

where

W = fractional window width = 0.25

w_L = two-sided loop noise bandwidth

T_s = symbol duration

and

$$\text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2} dt$$

Certainly the loop will be in lock as long as the phase error is such that symbol transitions remain within the fractional window width (i.e., $|\phi| < \pi/4$ for $W = 0.25$). This is a conservative lock definition since the loop applies restoring force for transitions outside the fractional window width. Thus the loop probably has a longer average time to loss of lock than that predicted by this analysis.

Let q be the probability that the phase error is within the fractional window width on a particular transition. Then

$$\begin{aligned} q &= P(|\phi| < \pi/4) \\ &= 2 \int_0^{\pi/4} \frac{1}{\sqrt{2\pi} \sigma} \exp[-\phi^2/2\sigma^2] d\phi \\ &= 2 \int_0^{\pi/4\sigma} \frac{1}{\sqrt{2\pi}} \exp[-x^2/2] dx \end{aligned}$$

$$= 2 \left\{ 0.5 - \int_{\pi/4\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} \exp[-x^2/2] dx \right\} \quad (3)$$

Thus

$$q = 1 - 2Q(\alpha)$$

where

$$\begin{aligned} Q(\alpha) &= \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} \exp[-x^2/2] dx \\ &\cong \frac{1}{\sqrt{2\pi} \alpha} \exp[-\alpha^2/2] \end{aligned} \quad (4)$$

for $\alpha = \pi/4\sigma > 2$ (Ref. 3).

Consider a series of transitions separated by a time interval T long enough so that the phase error at one transition may be considered independent of that at the previous transition. Roughly, this requires $T \geq 1/w_L$, where w_L is the two-sided loop noise bandwidth. The average number of transitions separated by T for which the phase error is within the fractional window width is then

$$\bar{n} = \sum_{n=1}^{\infty} n q^n = \frac{q}{1-q^2} \quad (5)$$

Substituting $q = 1 - 2Q(\alpha)$ yields

$$\bar{n} = \frac{1 - 2Q(\alpha)}{4Q^2(\alpha)} \cong \frac{1}{4Q^2(\alpha)}, \quad Q(\alpha) \ll 1 \quad (6)$$

This corresponds to an average time interval

$$T_{\text{ave}} = \bar{n}T = \frac{\bar{n}}{w_L} = \frac{\bar{n}T_s}{w_L T_s} \quad (7)$$

where T_s is the symbol duration.

III. Example

For 2-kbps (32,6) block coded data,

$$T_s = \left(\frac{6}{32} \right) \left(\frac{1}{2 \times 10^3} \right) \cong 94 \mu\text{s}$$

The values of α , $Q(\alpha)$, and \bar{n} as a function of loop signal-to-noise ratio are given in Table 1 as well as T_{ave} for relative loop bandwidths of 0.02 and 0.005.

From Table 1 it appears that a reasonable threshold is a loop signal-to-noise ratio of about 12 dB. Using Eq. (2), it can be shown that this corresponds to $ST_s/N_0 \cong 0$ dB for $w_L T_s = 0.02$ and $ST_s/N_0 \cong -3$ dB for $w_L T_s = 0.005$.

For a (32,6) block code, $ST_b/N_0 = ST_s/N_0 + 7.3$ dB. Thus, the threshold corresponds to $ST_b/N_0 = 7.3$ and 4.3 dB for $w_L T_s = 0.02$ and 0.005, respectively, which agrees very well with the experimental results.

Acknowledgment

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References

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Table 1. T_{ave} versus SNR_L

SNR_L , dB	$\alpha = \frac{\pi}{4\sigma}$	$Q(\alpha)$	\bar{n}	T_{ave}	
				$w_L T_s = 0.02$	$w_L T_s = 0.005$
11	2.79	2.92×10^{-3}	2.94×10^4	2.3 min	9.2 min.
12	3.13	9.51×10^{-4}	2.76×10^5	21.6 min	86.4 min
13	3.51	2.40×10^{-4}	4.34×10^6	5.7 h	22.8 h
14	3.94	4.31×10^{-5}	1.35×10^8	7.3 days	29.2 days